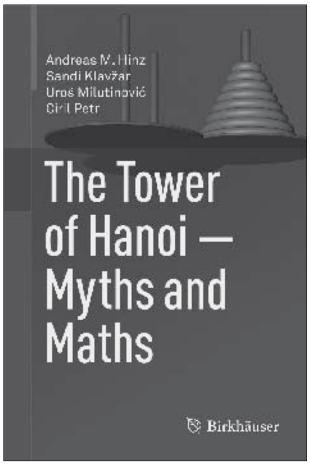
Book Reviews



Andreas M. Hinz, Sandi Klavžar, Uroš Milutinović and Ciril Petr

The Tower of Hanoi – Myths and Maths

Springer Basel, Basel 2013
xiv, 335 p.
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Reviewer: Jean-Paul Allouche

Three pegs, N discs, a tower, the Divine Rule, an anagram. The reader will have recognised the Hanoi Tower puzzle. Invented in 1883 by the French mathematician Lucas, under the anagram N. Claus, the Tower of Hanoi (also called the Tower of Brahma) consists of three pegs and N discs of sizes $1, 2, \ldots, N$. At the beginning of the game, the discs are on the same peg, in decreasing order with the largest disc below. At each step, the disc on the top of a peg can be moved to another peg, provided that the "divine rule" is respected: no disc can be put on a smaller one. At first view this puzzle seems to be one of those "mathematical" puzzles without real difficulty and/or interest. But this is absolutely not the case! In this nice book, the authors show a large number of mathematical results coming from or associated with the Tower of Hanoi.

If one thinks of computer science, the Tower of Hanoi is "the" classical example of the possibility of transforming recursive algorithms into iterative algorithms.

The reader might try to find "simple" (or "easy to write") algorithms to solve the Hanoi puzzle and then try to concoct an optimal solution (i.e., with a minimal number of moves). Another computer science flavoured result is the occurrence of a 2-automatic sequence describing the moves for N discs (for N integer or infinity). This result is a bridge towards number theory (recall that Lucas was a famous number theorist); binary expansions, integer sequences (e.g. the *ruler* sequence, also known as the Gros sequence), square-free sequences, fixed points of morphisms of the free monoid on two letters, the Pascal triangle, the Stern sequence, etc. show up in the detailed study of the Hanoi Tower. Number theory is not the only field touched by this puzzle. One can find links with graph theory, topology, fractals (e.g., the Sierpiński curve), etc. More unexpectedly, by looking at generalised Hanoi puzzles, one can find chemistry (the number of Kekulé structures of a specific class of benzenoid hydrocarbons with the molecular formula $C_{12n+2}H_{6n+4}$) and … psychology (with Shallice's Tower of London and Ward-Allport's Tower of Oxford).

Each time I open the book I discover a renewed interest in the Tower of Hanoi. I am sure that this will be the case for all readers, who will share my enthusiasm and enjoy all of the chapters, not to mention the numerous exercices, the bibliography with 352 references and the 21 conjectures or open problems listed at the end of the book.

Alex Oliver, and Timothy Smiley

Plural Logic

Oxford University Press, Oxford 2013
xiv, 336 p.
ISBN 978-0-19-957042-3

Reviewer: Louis F. Goble

The Newsletter thanks Zentralblatt MATH and Louis F. Goble for the permission to republish this review, originally appeared as Zbl 1273.03002.

A short summary review like this cannot convey how interesting and important this book is. If it had been written a hundred years ago, the course of philosophy of logic, philosophy of mathematics, philosophy of language, not to mention formal logic itself, would have been quite different, and far more wholesome. As the title declares, the authors here present a logic of plurals, expressions that typically denote more than one individual, though they might denote merely single things, or nothing at all. These include proper names, e.g., 'British Isles'; definite descriptions, 'the authors of *Principia Mathematica*'; lists, 'Whitehead and Russell'; functional value terms, 'the wives of Henry VIII', '$\sqrt{4}$', 'the square roots of 4'; demonstratives, 'these', 'those'; etc. Such terms are ubiquitous in ordinary discourse, and essential too to mathematics and other formal disciplines. They become especially crucial to contexts of collective, as opposed to distributive, predication, e.g., 'Whitehead and Russell wrote *Principia Mathematica*', 'the premises entail the conclusion'. Yet plurals have been excluded from modern classical logic since its inception. This book provides a comprehensive corrective.

The book develops in three stages. The first, Chapters 2–4, presents the need for a rigorous and robust plural logic, arguing the inadequacy of alternative treatments of plural terms, such as analyses that 'change the subject' to